Critical correlators of three-dimensional gauge theories at finite temperature: exact results from universality

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According to the Svetitsky-Yaffe conjecture, a three-dimensional gauge theory undergoing a continuous deconfinement transition is in the same universality class as a two-dimensional statistical model with order parameter taking values in the center of the gauge group. This allows us to use conformal field theory techniques to evaluate exactly various correlation functions at the critical point. In particular, we show that the plaquette operator of the gauge theory is mapped into the energy operator of the dimensionally reduced model. The plaquette expectation value in presence of static sources for three-dimensional $SU(2)$ and $SU(3)$ theories at the deconfinement temperature can be exactly evaluated, providing some new insight about the structure of the color flux tube in mesons and baryons.

Universality applied to the finite temperature deconfinement transition of gauge theories takes the form of the Svetitsky-Yaffe conjecture [1]: a $(d+1)$-dimensional gauge theory with a second order finite temperature deconfinement transition is in the same universality class as the d-dimensional spin model with symmetry group of the order parameter given by the center of the gauge group (provided this spin model has a second order phase transition as well). The aim of this work is to exploit universality arguments to investigate the physics of the deconfinement transition. The key point is that correlation functions at criticality are universal: when the Svetitsky-Yaffe conjecture applies, we can predict that the gauge theory critical correlators will coincide with the ones of the spin model.

Now consider a $(2+1)$-dimensional gauge theory. The equivalent spin model will be described, at the critical point, by a two-dimensional conformal field theory, whose correlation functions are in principle known exactly. Therefore universality gives us access to exact critical correlators for $(2+1)$-dimensional gauge theories at the deconfinement point.

*presented by P. Provero

Let us focus on an explicit example, $(2+1)$-dimensional $SU(2)$ gauge theory, which the Svetitsky-Yaffe conjecture predicts to be in the same universality class as the two-dimensional Ising model. The values of the critical indices for the gauge theory deconfinement transition have in fact been shown to coincide to high accuracy with the known ones of the Ising model[2].

To compute critical correlators of the gauge theory, we need a mapping between observables of the gauge theory and of the Ising model. The first item in this mapping is the correspondence between the respective order parameters: the Polyakov loop will correspond to the spin operator of the Ising model.

Consider now the plaquette operator. Symmetry considerations suggest to assume that the plaquette operator is mapped into a linear combination of the identity and energy operators in the Ising model conformal algebra:

$$\Box \rightarrow c_1 1 + c_\epsilon \epsilon \qquad (1)$$

In principle, one should add all the secondary fields belonging to the conformal families of the 1 and ϵ operators, whose contributions are expected to disappear at large distances. Eq. (1) contains

the two leading terms of the operator product expansion of the plaquette in terms of local fields of the Ising model.

The simplest correlator to be studied to verify our assumption is just the plaquette expectation value, or rather its finite size scaling behavior. We consider a $(2+1)$-dimensional rectangular lattice of sides L_1, L_2, L_t with $L_t \ll L_1, L_2$ and periodic boundary conditions on all directions. Then we tune the coupling β to the critical value $\beta_c(L_t)$ (precise estimates are available in the literature [2]), and we measure the plaquette expectation value $\Box(L_1, L_2)$ as a function of the lattice sides L_1 and L_2.
Universality gives us the following prediction for this quantity:

$$\Box(L_1, L_2) = c_1 + c_\epsilon \frac{F(\tau)}{\sqrt{L_1 L_2}} + O\left(\frac{1}{L_1 L_2}\right) \qquad (2)$$

Here c_1 and c_ϵ are non universal constants whose value will depend on L_t and on the kind of plaquette we consider (timelike or spacelike). The coefficient of c_ϵ in Eq.(2) is the energy expectation value for the two-dimensional Ising model on a torus at the critical point (see *e.g.* [3]); $\tau = i\frac{L_1}{L_2}$ is the modular parameter of the torus, and $F(\tau)$ is expressed in terms of Dedekind η and Jacobi θ functions (see [4] for the explicit expression). The $O(1/L_1 L_2)$ term comes from secondary fields: Eq.(2) will be valid for asymptotically large lattices.
To verify the correctness of this prediction, we need to include all the data, coming from lattices of different shapes, in a single fit. Let us therefore define an "effective area" that incorporates the τ dependence predicted by the Ising model:

$$\alpha(L_1, L_2) = \frac{L_1 L_2}{F^2\left(i\frac{L_1}{L_2}\right)} \qquad (3)$$

Eq. (2) predicts a linear dependence of $\Box(L_1, L_2)$ on $1/\sqrt{\alpha}$, which is indeed well realized by the data, shown in Fig. 1. The data plotted in the figure come from lattices with aspect ratio L_1/L_2 between 1 and 3 (see Ref.[4] for details). Our assumption Eq. (1) is thus verified. The same analysis was performed in Ref.[5] for Z_2 gauge theory.

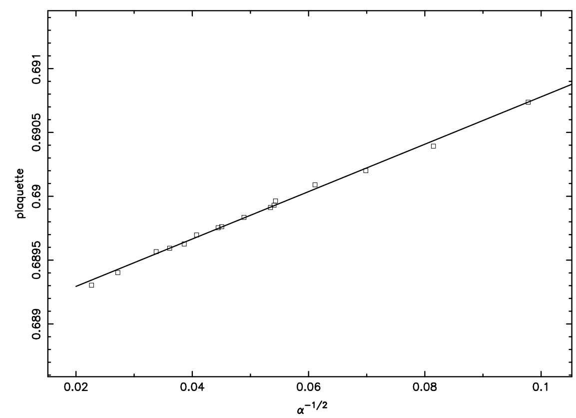

Figure 1. Time–like plaquette expectation value as a function of $\alpha^{-1/2}$ for $N_t = 2$

Suppose we want to study the spatial distribution of the color flux tube at the deconfinement point in a static meson, that is in presence of two static color sources. The correlator to be studied in $SU(2)$ theory is

$$\begin{aligned} G(x, x_1, x_2) &= \langle \Box(x) P(x_1) P(x_2) \rangle \\ &- \langle \Box \rangle \langle P(x_1) P(x_2) \rangle \end{aligned} \qquad (4)$$

where P is the Polyakov loop operator. Universality tells us that at the critical point

$$G(x, x_1, x_2) \propto \langle \epsilon(x) \sigma(x_1) \sigma(x_2) \rangle_{Ising} \qquad (5)$$

The Ising correlator on the r.h.s. is easily computed in conformal field theory and gives

$$G(x, x_1, x_2) \propto \frac{|x_1 - x_2|^{3/8}}{\left(|x - x_1|\,|x - x_2|\right)^{1/2}} \qquad (6)$$

which is represented in Fig.2.

Also $(2+1)$-dimensional $SU(3)$ gauge theory can be studied within the same approach. The Svetitsky-Yaffe conjecture predicts the universality class to be the one of the two-dimensional three state Potts model, whose critical point is described by a minimal conformal field theory with

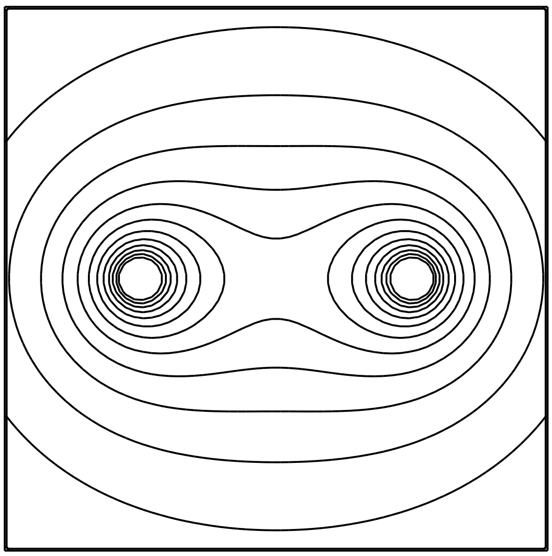

Figure 2. Color flux tube in a static meson

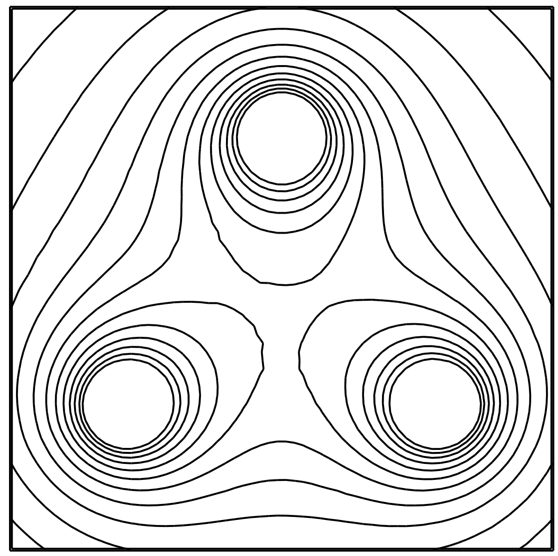

Figure 3. Color flux tube in a static baryon

central charge $c = 4/5$.
In $SU(3)$ we can construct a static baryon by considering three static color sources. The color flux tube distribution is then given by the correlator:

$$G(x, x_1, x_2, x_3) = \langle \Box(x) P(x_1) P(x_2) P(x_3) \rangle \\ - \langle \Box \rangle \langle P(x_1) P(x_2) P(x_3) \rangle \qquad (7)$$

which at the deconfinement point translates into a four point correlator of the two-dimensional conformal theory:

$$G(x, x_1, x_2, x_3) \propto \langle \epsilon(x) \sigma(x_1) \sigma(x_2) \sigma(x_3) \rangle_{c=4/5} \quad (8)$$

This can be computed exactly and is expressed in terms of hypergeometric functions: We refer the reader to Ref.[5] for the analytical expression and here we just show a plot of the flux tube distribution when the three static sources are at the vertices of an equilateral triangle, Fig. 3.

In conclusion, two main lessons can be learned from our results:

- Universality arguments provide a powerful, analytical approach to the non-perturbative physics of many interesting gauge theories.
- This is especially true for $(2+1)$-dimensional gauge theories, since critical behavior in two dimensions is completely understood with the methods of conformal field theory.

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